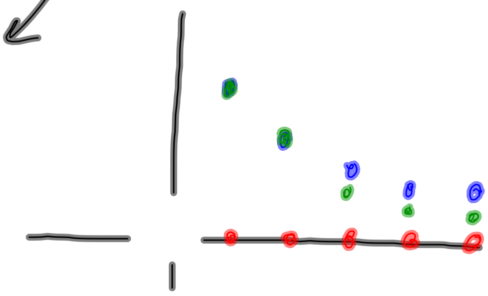
43.

$$\lim_{n\to\infty} \frac{1}{n!} = 0$$ converges to 0

$$0 \le \frac{1}{n!} \le \frac{1}{n}$$

$$\to 0$$



Jan 20-7:56 AM

8.2 L'Hopital's Rule

$$\boxed{\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} \quad \text{if the original limit is } \frac{0}{0} \text{ or } \frac{\infty}{\infty}}$$

{indeterminate form}

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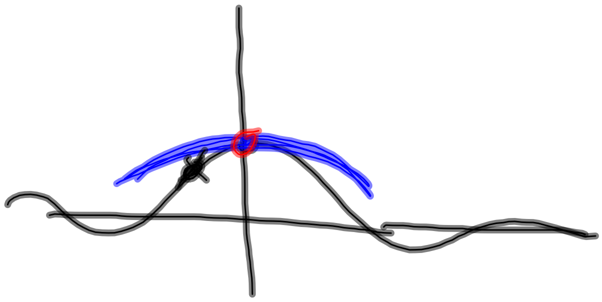
$$\lim_{x\to 0}\frac{\sin(x)}{x} = \quad \frac{\to 0}{\to 0} \qquad \lim_{x\to 0}\frac{\cos x}{1} = 1$$

graph sin(x)/x and cos(x)/1. How does this support l'hopital's rule?



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$$\lim_{x\to 0}\frac{\sqrt{1+x}}{x} = \frac{\to 1}{\to 0} = \infty \quad \{\text{no limit}\}$$

don't use l'hopital's rule not $\frac{0}{0}, \frac{\infty}{\infty}$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1-x/2}{x^2} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}-0-\frac{1}{2}}{2x} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{-\frac{1}{4}(1+x)^{-3/2}}{2} = -\frac{1}{8}$$

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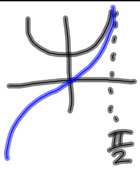
$\frac{\infty}{\infty}$

$$\lim_{x\to\pi/2}\frac{\sec x}{1+\tan x} = \cancel{\frac{\infty}{\infty}} = 1$$



$\frac{\pi}{2}$

$$\lim_{x\to\frac{\pi}{2}} \frac{\cancel{\sec x}\tan x}{\sec^{\cancel{2}} x}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\frac{\sin x}{\cancel{\cos x}}}{\frac{1}{\cancel{\cos x}}}\cdot\frac{\cos x}{1}$$

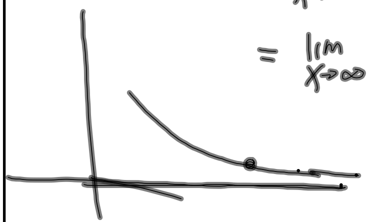
$$\lim_{x\to\infty}\frac{\ln x}{2\sqrt{x}} = \lim_{x\to\infty}\frac{\frac{1}{x}}{\cancel{2}\cdot\frac{1}{\cancel{2}}x^{-\frac{1}{2}}}$$

$\ln x \to \infty$, $2\sqrt{x} \to \infty$

$$= \lim_{x\to\infty}\frac{1}{x}\sqrt{x} \qquad \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty}\frac{\frac{1}{2}x^{-\frac{1}{2}}}{1} = \lim_{x\to\infty}\frac{1}{2\sqrt{x}}$$

$$= 0$$



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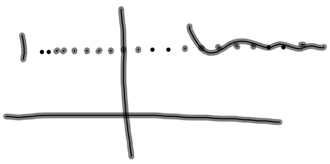
$\infty \cdot 0$ indet.

$$\lim_{x\to\infty} x\sin\frac{1}{x} = 1$$



$$\lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\frac{1}{x}} \qquad \frac{0}{0}$$

$$\lim_{x\to\infty}\frac{\cos\frac{1}{x}\cdot\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}} = 1$$

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$\infty - \infty$

$$\lim_{x\to1}\left(\frac{1(x-1)}{\ln x(x-1)}-\frac{1}{(x-1)}\frac{\ln x}{\ln x}\right)= \quad \lim_{x\to1}\frac{x-1-\ln x}{\ln x\,(x-1)} \quad \frac{0}{0}$$

$\ln 1 = 0$

$$\lim_{x\to1}\frac{\left(1-\frac{1}{x}\right)}{\left(\ln x\cdot 1+(x-1)\frac{1}{x}\right)}\quad\frac{x}{x}$$

$$\lim_{x\to1}\frac{x-1}{x\ln x + x-1}$$

$$\lim_{x\to1}\frac{1}{x\cdot\frac{1}{x}+\ln x + 1} = \frac{1}{2}$$

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1^∞ indet.

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = y \quad \text{set} = \text{to } y \qquad \begin{matrix}2.71828\ldots\\ e\end{matrix}$$

$$\lim_{x\to\infty}\ln\left(1+\frac{1}{x}\right)^x = \ln y \qquad \text{take ln of both sides}$$

$$\lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right) = \ln y$$

$$\frac{0}{0}\quad \lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \ln y$$

$$\lim_{x\to\infty}\frac{\frac{1}{\left(1+\frac{1}{x}\right)}\cdot\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = \ln y$$

$$1 = \ln y$$

$$e = y$$

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